THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Applied Mathematics

Lightboard Project

3. Continuity and Intermediate Value Theorem

3.1 Show that the equation $3x^3 - |\sin(x)| + \cos(x) = 1.5$ has a solution in the interval [-1,1]. [21222 Exam]

3.2 Let

$$f(x) = \begin{cases} \ln(3^x + 10x + x^3) - 5, & \text{if } 0 \le x \le 5, \\ be^{|2-x|}, & \text{if } 5 \le x \le 7. \end{cases}$$

(i) Find the value of b such that $f(x)$ is continuous at $x = 5$.

(ii) With the value of b in the previous question, show that the equation $f(x) = 0$ has a solution in the interval $[0, 7]$. [22231 Test2]

3.3 Let

$$f(x) = \begin{cases} x^2, & \text{if } x \le 1, \\ 2 + \dfrac{a\sin(2x)}{x}, & \text{if } x > 1. \end{cases}$$

(a) Find the value of a so that the function $f(x)$ is continuous at $x = 1$.

(b) With the value of a in the previous question, show that the equation $f(x) = 2$ has a solution in the interval $(0, 2)$. Justify your steps and answer. [19202 Test2]

3.4 Let

$$f(x) = \begin{cases} \frac{2}{3}\sin(3x) + 5, & \text{if } x < 0, \\ e^{2x} + a, & \text{if } x \ge 0. \end{cases}$$

(a) Find the value of a so that the function $f(x)$ is continuous at $x = 0$.

(b) With the value of a in (a), show that the equation $f(x) = 10$ has EXACTLY one solution in the interval $(0, 1)$. Justify your steps and answer. [16172 Exam]

3.5 Show that the equation $\dfrac{1}{x} = 4\cos(x)$ has a solution in some subset of the interval $(0, \frac{\pi}{2})$. [18192 Test2]