THE HONG KONG POLYTECHNIC UNIVERSITY

Department of Applied Mathematics

Lightboard Project

5. One-Sided Derivatives and Differentiability

5.1 Let

$$f(x) = \begin{cases} a\sin(3x) + 5 & \text{if } x < 0, \\ e^{2x} + b & \text{if } x \geq 0. \end{cases}$$

Find the unknown values of a and b such that $f(x)$ is continuous and differentiable at $x = 0$. [16172 Exam]

5.2 Let

$$f(x) = \begin{cases} x^2 + bx + c & \text{if } x \leq 0, \\ \dfrac{\ln(1+x)}{x} & \text{if } x > 0. \end{cases}$$

If $f'(0)$ exists, find b. [14151 Exam]

5.3 Let

$$f(x) = \begin{cases} \dfrac{ax}{2} + \dfrac{5}{2} & \text{if } x < 1, \\ b + \sqrt{x} & \text{if } x \geq 1. \end{cases}$$

Assuming f is continuous at $x = 1$, calculate $\lim\limits_{h\to 0^-} \dfrac{f(1+h) - f(1)}{h}$ and $\lim\limits_{h\to 0^+} \dfrac{f(1+h) - f(1)}{h}$. Determine the value of a and b so that f is differentiable at $x = 1$. [17182 Test2]

5.4 Let

$$f(x) = \begin{cases} e^{3x} + a\cos(x) & \text{if } x < 0, \\ b + \sin(bx) & \text{if } x \geq 0. \end{cases}$$

Assuming f is continuous at $x = 0$, calculate $\lim\limits_{h\to 0^-} \dfrac{f(h) - f(0)}{h}$ and $\lim\limits_{h\to 0^+} \dfrac{f(h) - f(0)}{h}$. Determine the value of a and b so that f is differentiable at $x = 0$. [18191 Test2]

5.5 Let

$$f(x) = \begin{cases} ax \sin \dfrac{1}{x^2} - x & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ \arctan(\dfrac{1}{x}) + b & \text{if } x < 0. \end{cases}$$

Find the value of a and b such that $f(x)$ is differentiable everywhere and compute $f'(x)$. [You are required to express the one-sided derivative by the first principle of differentiation.] [21221 Exam]